

Math Wrangle
Marin Math Circle

October 26, 2011

1. Drake takes three minutes to descend the down escalator standing still. Josh walks down the same escalator in one minute, taking 120 steps in transit. If each step is 20 cm high, then what is the overall height of the escalator?
2. A sequence a_1, a_2, a_3, \dots of real numbers satisfies $a_{n+1} = a_n + a_{n+3}$ for all $n \geq 1$. Find, with proof, the maximum possible number of consecutive positive terms that may occur in such a sequence.
3. Draw a convex polygon with n sides. Start connecting its vertices with diagonals, and when you can't draw any more without intersecting existing diagonals, count the total number. Find a formula for the total number of diagonals and for the total number of triangles formed, and show that these formulas do not depend on the manner in which the diagonals are drawn at each step.
4. The length of one side of a triangle is 3.8 cm and the length of another side is 0.6 cm. Find the length of the third side, if it is known that it is a whole number of cm.
5. Is it possible to find a number of the form $11 \dots 1100 \dots 00$ that is divisible by 2011? Prove or disprove.
6. Consider the collection of all three-element subsets of $\{1, 2, 3, \dots, n\}$. Prove that there are as many subsets in which one of the elements equals the sum of the other two as there are subsets in which one of the elements is the average of the other two.
7. An equilateral triangle ABC is given. Find the locus of points M such that both triangles ABM and ACM are isosceles.
8. Savannah draws diagonals within some of the unit squares of an 8×8 chessboard so that none of the diagonals intersect, even at their endpoints. Find, with proof, the maximum number of diagonals that Savannah can draw.