

Math Wrangle, Round 1

1. Each day at noon, a mail steamer leaves Savannah for Belfast, while a steamer of the same line leaves Belfast for Savannah. Each steamer spends exactly 7 (24-hour) days at sea, and travel along the same route. How many steamers of the same company will the Belfast-Savannah steamer meet while underway?
2. Consider the set of points that are inside or within one unit of a rectangular parallelepiped (box) that measures 3 by 4 by 5 units. What is the volume of the set?
3. Simplify the fraction:
$$\frac{1 \cdot 2 \cdot 3 + 2 \cdot 4 \cdot 6 + 4 \cdot 8 \cdot 12 + 7 \cdot 14 \cdot 21}{1 \cdot 3 \cdot 5 + 2 \cdot 6 \cdot 10 + 4 \cdot 12 \cdot 20 + 7 \cdot 21 \cdot 35}$$
4. Using a pencil, ruler, and sheet of graph paper, how can you draw a square with area (a) double and (b) 5 times larger than the area of one square?
5. Which is greater: the sum of the lengths of the sides of a quadrilateral or the sum of the lengths of its diagonals?
6. Will it be possible to write more than 50 different two-digit numbers on a blackboard so that there won't be two numbers on the board that total 100?
7. Is it possible to fill each square of an 8 x 8 chessboard with one of the numbers 1, 2, 3 in such a way that when you sum each row, each column, and each of the two diagonals you get 18 different sums?
8. A straight bar 2 meters in length is cut into 5 pieces, each no less than 17 cm long. Prove that amongst these pieces there are three that can be put together to form a triangle.