

### Tilings with dominos, straight, bent and triangular triominoes

1. In how many different ways can a rectangular  $2 \times n$  board be tiled with dominos? Start with small numbers  $n$ , then generalize.

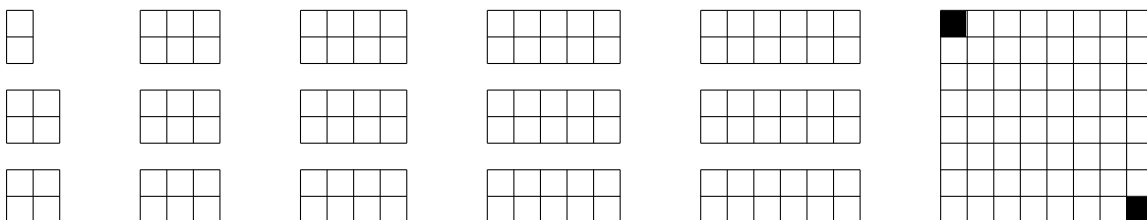
2. Is it possible to tile a  $5 \times 5$  square board with dominos?



3. Is it possible to tile with dominos a  $5 \times 5$  square board from which one square has been removed? Does it matter which one has been removed?

4. Is it possible to tile with dominos an  $8 \times 8$  rectangular board from which two opposite corner have been removed?

5. Find all squares on an  $8 \times 8$  rectangular board such that if one of these squares is removed, then the remaining part can be tiled with (straight) triominoes.



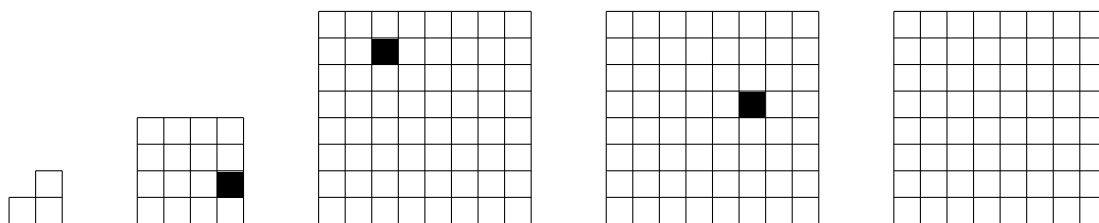
A bent triomino is of a  $2 \times 2$  board from which one corner has been removed.




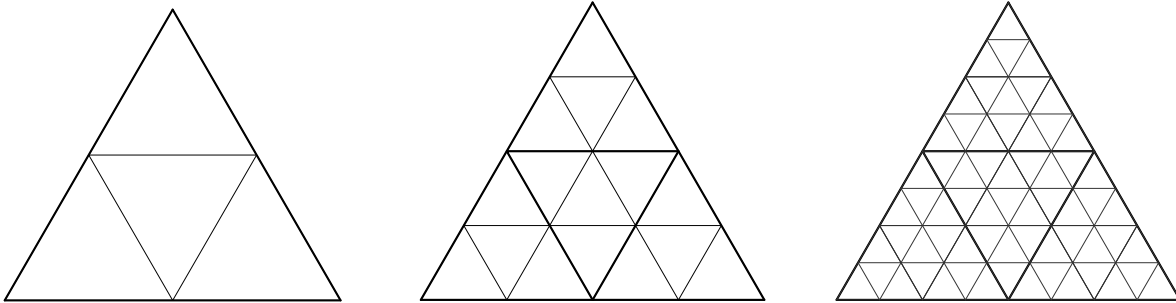
6. Is it possible to tile an  $2^n \times 2^n$  board from which one square has been removed with bent triominoes? Does it matter which square has been removed?

7. Which square boards of size  $n \times n$  from which one square has been removed can be tiled with bent triominoes? (Hint: The  $5 \times 5$  board is special.)

8. Which  $n \times m$  rectangular boards can be tiled with bent triominoes?



Write  $T(n)$  for a triangular board of side-length  $2^n$  which is subdivided into equilateral triangles each of side length 1. If a triangle shares one (or two) of its sides with the large triangle, then it is called an *edge triangle*. If it shares two of its sides with the large triangle, then it is called a corner triangle. A (triangular) triomino is a *tile* consisting of three adjacent triangles. 



9. For which  $n$  is it possible to tile the remaining board with triangular triominoes after any (one) corner triangle is removed from  $T(n)$ ?
10. For which  $n$  is it possible to tile the remaining board with triangular triominoes after any (one) edge triangle is removed from  $T(n)$ ?
11. For which  $n$  is it possible to tile the remaining board with triangular triominoes after all the corner triangles and any other (one more) triangle are removed from  $T(n)$ ?
12. For which  $n$  is it possible to tile the remaining board with triangular triominoes after if any (one) triangle not adjacent to a corner triangle is removed from  $T(n)$ ?

