

Pythagorean Points

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1. If a point (x, y) is on the circle of radius 1 centered at the origin, then $x^2 + y^2 = 1$. Why?
2. Take the point $(3, 4)$ and reflect it in
 - a) the y -axis.
 - b) the x -axis.
 - c) the line $x = 5$.
 - d) the line $y = -1$.
 - e) the line $y = x$.
 - f) the line $y = 3x$.
 - g) the line $y = x/2$.
3. Take the midpoint between
 - a) the points $(0, 0)$ and $(3, 4)$.
 - b) the points $(5, 0)$ and $(3, 4)$.
 - c) the points $(1, 0)$ and $(3/5, 4/5)$.
4. What is the other endpoint of the segment if
 - a) $(0, 0)$ is one endpoint and $(3, 4)$ is the midpoint?
 - b) $(1, 0)$ is one endpoint and $(3, 4)$ is the midpoint?
 - c) $(1, 0)$ is one endpoint and $(4/5, 2/5)$ is the midpoint?
5. If a , b , and c are all integers, what are the possible types of solutions you might find for $ax^2 + bx + c = 0$?
6. Draw the line from the point $(-1, 0)$ to a point on the line $x = 1$ where y is a whole number or a fraction. Where does that line intersect the circle?
Can you explain why every Pythagorean point can be constructed this way?
7. Choose a line through the origin whose slope is a fraction. What happens when you reflect $(1, 0)$ over that line?
Can you explain why every Pythagorean point can be constructed this way?
8. How does all this change if we consider the circle $x^2 + y^2 = 2$ instead?
9. What about $x^2 + y^2 = 3$?
10. What about $y^2 = x^3 + 17$?