XV. AMC 8 Practice Questions
04-01

- On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

(A) 6 **(B)** 102 **(C)** 204 **(D)** 864 **(E)** 1224

2004 AMC 8, Problem #1
"How many kilometers does 1 centimeter represent?"

- **Solution**

(B) If 12 centimeters represents 72 kilometers, then 1 centimeter represents 6 kilometers. So 17 centimeters represents $17 \times 6 = 102$ kilometers.

Difficulty: Easy

NCTM Standard: Measurement:
apply appropriate techniques, tools, and formulas to determine measurements

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Directly Proportional

- Aunt Anna is 42 years old. Caitlan is 5 years younger than Briana, and Brianna is half as old as Aunt Anna. How old is Caitlan?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 21 **(E)** 37

2000 AMC 8, Problem #1— "Brianna is half as old as Aunt Anna"

- **Solution**

(B) Brianna is half as old as Aunt Anna, so Brianna is 21 years old. Caitlan is 5 years younger than Brianna, so Caitlan is 16 years old.

Difficulty: Easy

NCTM Standard: Number and Operations Standard: Understand meanings of operations and how they relate to one another

Mathworld.com Classification:
Algebra > General Algebra > Algebra
Number Theory > Arithmetic

- Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they share, how many meals should they have ordered to have just enough food for the 12 of them?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 15 **(E)** 18

2004 AMC 8, Problem #3
"Find the ratio of food to people."

- **Solution (A)** If 12 people order $\frac{18}{12} = 1\frac{1}{2}$ times too much food, they should have ordered $\frac{12}{\frac{3}{2}} = \frac{2}{3} \times 12 = 8$ meals.

OR

Let x be the number of meals they should have ordered. Then,

$$\frac{12}{18} = \frac{x}{12},$$

so

$$x = 8.$$

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: Understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Ratio

- How many different combinations of \$5 bills and \$2 bills can be used to make a total of \$17? Order does not matter in this problem.

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2002 AMC 8, Problem #2—"Can the number of \$5 bills be even?"

- **Solution (A)** Since the total \$17 is odd, there must be an odd number of \$5 bills. One \$5 bill plus six \$2 bills is a solution, as is three \$5 bills plus one \$2 bill. Five \$5 bills exceeds \$17, so these are the only two combinations that work.

Difficulty: Medium-easy

NCTM Standard: Problem Solving Standard for Grades 6–8: Apply and adapt a variety of appropriate strategies to solve problems.

Mathworld.com Classification:
Number Theory > Diophantine Equations > Coin Problem

- Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

  The losing team of each game is eliminated from the tournament. If sixteen teams compete, how many games will be played to determine the winner?

(A) 4 **(B)** 7 **(C)** 8 **(D)** 15 **(E)** 16

2004 AMC 8, Problem #5
"How many teams need to lose in order for one team to be left?

- **Solution**

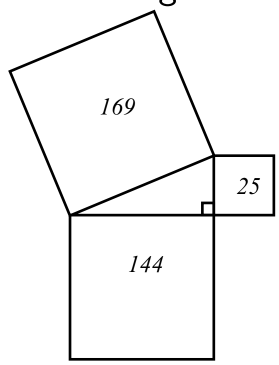
  (D) It takes 15 games to eliminate 15 teams.

Difficulty: Medium

NCTM Standard: Data Analysis and Probability
develop and evaluate inferences and predictions that are based on data

Mathworld.com Classification:
Discrete Mathematics > Graph Theory > Directed Graph > Tournament

- Given the areas of the three squares in the figure, what is the area of the interior triangle?



(A) 13 **(B)** 30 **(C)** 60 **(D)** 300 **(E)** 1800

2003 AMC 8, Problem #6— "What are the side lengths of the triangles?"

- **Solution**
  (B)

$$A = \frac{1}{2}(\sqrt{144})(\sqrt{25})$$
$$A = \frac{1}{2} \cdot 12 \cdot 5$$
$$A = 30 \text{ square units}$$

Difficulty: Medium

NCTM Standard: Geometry Standard: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square

- Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't talk any more but leaves the phone on, how many more hours will the battery last?

(A) 7 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** 15

2004 AMC 8, Problem #12
"The phone has been used for 1 hour to talk, how much of the battery has it used?"

- **Solution**

(B) The phone has been used for 60 minutes, or 1 hour, to talk, during which time it has used $\frac{1}{3}$ of the battery. In addition, the phone has been on for 8 hours without talking, which used an additional $\frac{8}{24}$ or $\frac{1}{3}$ of the battery. Consequently, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ of the battery has been used, meaning that $\frac{1}{3}$ of the battery, or $\frac{1}{3}\times 24=8$ hours remain if Niki does not talk on her phone.

OR

Niki's battery has 24 hours of potential battery life. By talking for one hour, she uses $\frac{1}{3}\times 24=8$ hours of battery life. In addition, the phone is left on and unused for 8 hours, using an additional 8 hours. This leaves $24-8-8=8$ hours of battery life if the phone is on and unused.

Difficulty: Medium-hard

NCTM Standard: Measurement Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Ratios

- Bicycle license plates in Flatville each contain three letters. The first is chosen from the set $\{C, H, L, P, R\}$, the second from $\{A, I, O\}$, and the third from $\{D, M, N, T\}$. When Flatville needed more license plates, they added two more letters. The new letters may be added to one set, or one letter may be added to one, and one to another set. What is the largest possible number of *additional* license plates that can be made by adding two letters?

(A) 24 **(B)** 30 **(C)** 36 **(D)** 40 **(E)** 60

1999 AMC 8, Problem #15— "How many license plates could originally be made? Where can the two letters be placed so the most new license plates will be created?"

- **Solution**

(D) Before new letters were added, five different letters could have been chosen for the first position, three for the second, and four for the third. This means that $(5)(3)(4) = 60$ plates could have been made.
If two letters are added to the second set, then $(5)(5)(4) = 100$ plates can be made. If one letter is added to each of the second and third sets, then $(5)(4)(5) = 100$ plates can be made. None of the other four ways to place the two letters will create as many plates. So, $100 - 60 = 40$ ADDITIONAL plates can be made.
Note: Optimum results can usually be obtained in such problems by making the factors as nearly equal as possible.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard: Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Permutations > Combination

AMC 8 Practice Questions Continued
00-21

- Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2000 AMC 8, Problem #21— "Make a complete list of equally likely outcomes."

- **Solution**

(B) Make a complete list of equally likely outcomes:

Keiko	Ephraim	Same Number of Heads?
H	HH	No
H	HT	Yes
H	TH	Yes
H	TT	No
T	HH	No
T	HT	No
T	TH	No
T	TT	Yes

The probability that they have the same number of heads is $\frac{3}{8}$.

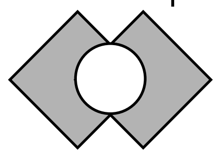
Difficulty: Hard

NCTM Standard: Data Analysis and Probability Standard: Understand and apply basic concepts of probability

Mathworld.com Classification: Probability and Statistics > Probability > Coin Tossing

AMC 8 Practice Questions Continued
04-25

- Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



(A) $16 - 4\pi$ **(B)** $16 - 2\pi$ **(C)** $28 - 4\pi$ **(D)** $28 - 2\pi$ **(E)** $32 - 2\pi$

2004 AMC 8, Problem #25
"Draw in the square that exists in the middle. What is its side length?

- **Solution**

(D) The overlap of the two squares is a smaller square with side length 2, so the area of the region covered by the squares is $2(4\times4)-(2\times2) = 32-4 = 28$. The diameter of the circle has length $\sqrt{2^2+2^2} = \sqrt{8}$, the length of the diagonal of the smaller square. The shaded area created by removing the circle from the squares is $28 - \pi\left(\frac{\sqrt{8}}{2}\right)^2 = 28 - 2\pi$.

Difficulty: Hard
NCTM Standard: Geometry
analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
Mathworld.com Classification:
Geometry > Plane Geometry > Circles > Diameter